AC-0213

M.A. / M. Sc. (PArt - I) Mathematics (External) Examination

April / May – 2015

Paper - 405 : Graph Theory & Discrete Structure

Time : Hours] [Total Marks :

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M.A. / M. Sc. (PArt - I) (Mathematics) (External)**

Name of the Subject :

☛ **PAPER - 405 : GRAPH THEORY & DISCRETE STRUCTURE**

☛ Subject Code No. : **0 2 1 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions

(3) Figures to the right indicate marks.

(4) Follow the usual notations and conuentions.

1 Attempt any **FIVE**. [10]

1. Prove that an infinite graph with finite number of edges must have infinite number of isolated vertices.
2. Prove that a connected graph G remains connected after removing an edge e_i from G iff e_i is in some circuit in G.
3. Define edge connectivity and separable graph.
4. Prove that Hamiltonian path is a spanning tree.
5. Define atoms and antiatoms.
6. For an onto semigroup homomorphism $g: <S,*> \to <T,\Delta>$ if $e \in S$ is an identity then $g(e) \in T$ is an identity.
7. Prove that every chain is a distributive lattice.

2 (a) Prove that a graph G is disconnected iff its vertex set V can be partitioned into two nonempty, disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2. [5]

(b) Prove that a connected graph G is an Euler Graph iff it can be decomposed into circuits. [5]

(c) Prove that in a complete graph with n(≥3) vertices, there are (n-1)/2 edge disjoint Hamiltonian circuits where n is an odd number. [5]

OR

2 (a) Prove that a tree with n vertices has n-1 edges. [5]

(b) What is the minimum possible height of an n-vertex binary tree? [5]

(c) (i) Discuss the Utilities Problem. [5]
(ii) Show that an infinite graph with a finite number of vertices will have at least one pair of vertices joined by an infinite number of parallel edges

3 (a) Prove that the complete graph of two vertices is non-planar. [5]
(b) Prove that a graph can be embedded in a surface of the sphere iff it can be embedded in a plane. [5]
(c) Discuss the observation about the incidence matrix. [5]

OR

3 (a) Discuss the observation about the circuit matrix. [5]
(b) Discuss the observation about the cut-set matrix. [5]
(c) (i) In a connected graph G, prove that any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set. [5]
(ii) Prove that every circuit has an even number of edges in common with any cut-set.

4 (a) Define monoids and prove that for S = {p, q, r}, $< \rho(S), \cup >$ *and* $< \rho(S), \cap >$ are monoids. [5]
(b) Write a short note on mixed base number system. [5]
(c) State and prove the cancellation law of multiplication. [5]

OR

4 (a) Define cancatenation and prove that for any alphabet 'V', $<V^*, \cdot, \wedge>$ is a monoid. [5]
(b) Discuss different properties of a number system. [5]
(c) Prove that $<S_{210}, D>$ is a sublattice of $<I_+, D>$, where S_{210} is a set all the divisors of 210. Also give any example which justifies that any subset of L which is a lattice need not to be a sublattice. [5]

5 (a) Define lattice and sublattice. Also show that in a bounded, distributive lattice, the elements which have complements form a sublattice. [5]
(b) For the function$f = a + b + c$, give the circuit diagram and cube representation. [5]
(c) In a lattice show that [5]

$$(a * b) \oplus (c * d) \leq (a \oplus c) * (b \oplus d)$$
$$(a * b) \oplus (b * c) \oplus (c * a) \leq (a \oplus b) * (b \oplus c) * (c \oplus a)$$

OR

5 (a) Prove that the operations of meet and join satisfy the commutative, associative, idempotent and law of absorption. [5]
(b) Prove that $< L \times S, +, \cdot >$ is a lattice. [5]
(c) Define complemented lattice. Show that the De Morgan's laws hold in Complemented, distributive lattice. [5]